**MECHANICS (C) UNIT 1** **TEST PAPER 7**

Take $g = 9{\cdot}8$ ms$^{-2}$ and give all answers correct to 3 significant figures where necessary.

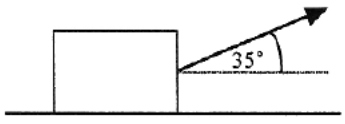
1. Briefly define the following terms used in modelling in Mechanics:
   (i) lamina, (ii) uniform rod, (iii) smooth surface, (iv) particle. [4]

2. A particle $P$ moves in a straight line so that its velocity $v$ ms$^{-1}$ at time $t$ seconds is given, for $t \geq 1$, by the formula $v = 2t + \frac{8}{t^2}$. Find the time when the acceleration of $P$ is zero. [4]

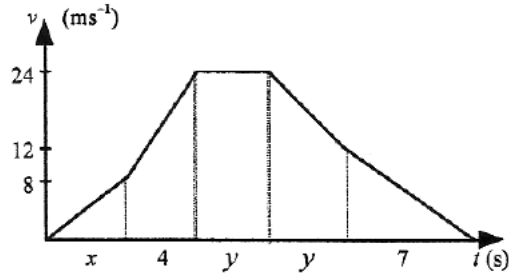
3. **F** and **G** are two forces. **F** has magnitude 15 N and acts on a bearing $\alpha$, where $\alpha < 90°$ and $\tan\alpha = \frac{3}{4}$. **G** has magnitude 13 N and acts on a bearing $\beta$, where $\beta < 90°$ and $\tan\beta = \frac{12}{5}$.
   The resultant of **F** and **G** is **R**.
   Calculate the magnitude of **R** and the bearing of the direction in which **R** acts. [6]

4. Two small smooth spheres $A$ and $B$, of equal radius but masses $m$ kg and $km$ kg respectively, where $k > 1$, move towards each other along a straight line and collide directly. Immediately before the collision, $A$ has speed 5 ms$^{-1}$ and $B$ has speed 3 ms$^{-1}$. After the collision, the speed of $B$ is 4 ms$^{-1}$.
   (i) Show that the speed of $A$ immediately after the collision is $(7k - 5)$ ms$^{-1}$ and deduce that the direction of $A$'s motion is reversed. [5]
   $B$ now receives a further impact in which the change in its momentum is $mu$ Ns, as a result of which a second collision between it and $A$ occurs.
   (ii) Show that $u > k(7k - 1)$. [4]

5. A string is attached to a packing case of mass 12 kg, which is at rest on a rough horizontal plane. When a force of magnitude 50 N is applied at the other end of the string, and the string makes an angle of 35° with the vertical as shown, the case is on the point of moving.




   (i) Find the coefficient of friction between the case and the plane. [5]
   The force is now increased, with the string at the same angle, and the case starts to move along the plane with constant acceleration, reaching a speed of 2 ms$^{-1}$ after 4 seconds.
   (ii) Find the magnitude of the new force. [5]
   (iii) State any modelling assumptions you have made about the case and the string. [2]

6. 


The velocity-time graph illustrates the motion of a particle which accelerates from rest to 8 $ms^{-1}$ in $x$ seconds and then to 24 $ms^{-1}$ in a further 4 seconds. It then travels at a constant speed for another $y$ seconds before decelerating to 12 $ms^{-1}$ over the next $y$ seconds and then to rest in the final 7 seconds of its motion.

Given that the total distance travelled by the particle is 496 m,

(i) show that $2x + 21y = 195$. [4]

Given also that the average speed of the particle during its motion is 15·5 $ms^{-1}$,

(ii) show that $x + 2y = 21$. [3]

Hence find

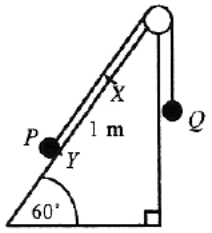
(iii) the values of $x$ and $y$, [2]

(iv) the acceleration for each section of the motion. [3]

7. $X$ and $Y$ are two points 1 m apart on a line of greatest slope of a smooth plane inclined at 60° to the horizontal. A particle $P$ of mass 1 kg is released from rest at $X$.

(i) Find the speed with which $P$ reaches $Y$. [4]

$P$ is now connected to another particle $Q$, of mass $M$ kg, by a light inextensible string. The system is placed with $P$ at $Y$ on the plane and $Q$ hanging vertically at the other end of the string, which passes over a small fixed pulley at the top of the plane.




The system is released from rest and $P$ moves up the plane with acceleration $\frac{g}{5}$.

(ii) Show that $M = \frac{5\sqrt{3}+2}{8}$. [7]

State a modelling assumption that you have made about the pulley. Briefly state what would be implied if this assumption were not made. [2]

# MECHANICS 1 (C) TEST PAPER 7 : ANSWERS AND MARK SCHEME

1. (i) 2-D rigid body (ii) 1-D rigid body, centre of mass at mid-pt. B1 B1
(iii) No frictional force (iv) Mass concentrated at a point B1 B1 4

2. $a = 2 - 16t^{-3} = 0$ when $t^3 = 8$ $t = 2$ M1 A1 M1 A1 4

3. Net force to east = 9 + 12 = 21 Net force to north = 12 + 5 = 17 B1 B1
Resultant = $\sqrt{(21^2 + 17^2)} = 27{\cdot}0$ N M1 A1
Direction (as bearing) = $\tan^{-1}(21/17) = 51{\cdot}0°$ M1 A1 6

4. (i) $5m - 3km = mv_A + 4km$ $\div m$ : $v_A = 5 - 7k$, $< 0$ as $k > 1$, so M1 A1 M1
speed of $A = (7k - 5)$ ms$^{-1}$ and direction is reversed A1 A1
(ii) $B$'s speed is now increased by $\frac{u}{k}$ and its direction changed, M1
so must have $\frac{u}{k} - 4 > 7k - 5$ $\frac{u}{k} > 7k - 1$ $u > k(7k - 1)$ M1 A1 A1 9

5. (i) Resolve : $R + 50 \sin 35° = 12g$, $50 \cos 35° = \mu R$ M1 A1 A1
$\mu(12g - 50 \sin 35°) = 50 \cos 35°$ $\mu = 0{\cdot}461$ M1 A1
(ii) Resolve : $R + F \sin 35° = 12g$, $F \cos 35° - \mu R = 12a$ M1 A1
$a = 0{\cdot}5$ : $F(\cos 35° + 0{\cdot}461 \sin 35°) = 6 + 0{\cdot}461(12g)$ $F = 55{\cdot}5$ B1 M1 A1
(iii) Case = particle (does not topple); string light and inextensible B1 B1 12

6. (i) Total dist. = sum of areas = $4x + 64 + 24y + 18y + 42$ M1 A1
Hence $4x + 42y + 106 = 496$ $2x + 21y = 195$ M1 A1
(ii) Total time = $x + 2y + 11$, so $496 = 15{\cdot}5(x + 2y + 11)$ M1 A1
$x + 2y + 11 = 32$ $x + 2y = 21$ A1
(iii) Solving simultaneously : $x = 3$, $y = 9$ M1 A1 (both)
(iv) $\frac{8}{3}, 4, 0, -\frac{4}{3}, -\frac{12}{7}$ ms$^{-2}$ B3 (–1 e.o.) 12

7. (i) Acc = $g \sin 60° = 8{\cdot}49$ ms$^{-2}$ $v^2 = 2as = 16{\cdot}97$ $v = 4{\cdot}12$ ms$^{-1}$ M1 A1 M1 A1
(ii) $T - g \sin 60° = a$, $Mg - T = Ma$ $a = \frac{g}{5}$ M1 A1 A1
Add : $Mg - g\frac{\sqrt{3}}{2} = M\frac{g}{5} + \frac{g}{5}$ $M(\frac{4g}{5}) = \frac{g}{5} + g\frac{\sqrt{3}}{2}$ M1 A1
x 10, $\div g$ : $8M = 2 + 5\sqrt{3}$ $M = \frac{5\sqrt{3}+2}{8}$. M1 A1
Assumed pulley is smooth. If not, tensions in two sections of string B1
are not equal B1 13